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Defect cores in nematic equilibria: interplay between temperature and geometry

Nematics liquid crystals are anisotropic liquids with long-range orientational ordering, making them popular working materials for display applications and biomedical research. Nematic textures often exhibit intricate defect patterns, the commonest examples being Schlieren textures with striking optical properties. We consider two model problems for illustrative purposes. We first consider a spherical droplet with radial boundary conditions, for low temperatures below the nematic-isotropic transition temperature. We work within the Landau-de Gennes theory and show that the radial-hedgehog solution, which is analogous to a degree +1-vortex solution in superconductivity, is the only admissible uniaxial solution in the low-temperature limit. This follows from a careful blow-up type analysis of the singularity profile at the centre of the droplet. We use the instability of the radial-hedgehog solution with respect to biaxial perturbations to conclude that energy minimizers must have biaxial defect cores for this model problem, for sufficiently low temperatures.

The second problem concerns a punctured spherical droplet with radial boundary conditions. We show that the radially symmetric solution is locally stable for all temperatures below the nematic-isotropic transition temperature. We prove that the radially symmetric solution is, in fact globally stable i.e. is the global energy minimizer in the low temperature limit for the punctured droplet, in contrast to the instability result for a spherical droplet above.