Effect of a micro-structure on the thermal striping damage of an edge-cracked slab

D.J. Colquitt[1]

M.J. Nieves[2] N.V. Movchan[1] I.S. Jones[2] A.B. Movchan[1]

[1]Department of Mathematical Sciences
University of Liverpool

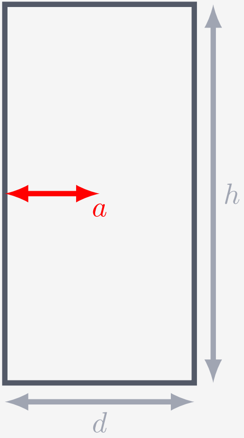
[2]School of Engineering

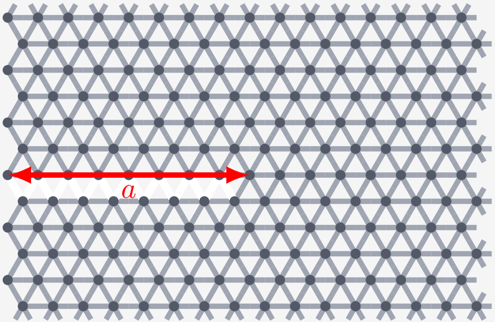
John Moores University

IUTAM Symposium on
Fracture Phenomena in Nature & Technology

DJC acknowledges the support of an EPSRC research studentship (grant EP/H018514/1)
MJN & ISJ acknowledge the financial support of EPSRC grant number EP/H018239/1
ABM & NVM acknowledge the financial support of the European Community's Seven Framework Programme under contract number PIAPP-GA-284544-PARM-2.

The thermally striped geometries





The uncoupled thermoelastic problem

In the continuum

$$\begin{aligned} \mathcal{L}\boldsymbol{U}(\boldsymbol{x};t) &= \alpha(3\lambda+2\mu)\nabla T(\boldsymbol{x};t), & \boldsymbol{x} &\in \Omega \setminus M_a, \\ \boldsymbol{\sigma}^{(\boldsymbol{n})}[\boldsymbol{U}](\boldsymbol{x};t) &= \alpha(3\lambda+2\mu)\boldsymbol{n}T(\boldsymbol{x};t), & \boldsymbol{x} &\in B_0 \cup B_d \cup M_a^+ \cup M_a^-, \\ \boldsymbol{U}(\boldsymbol{x};t) &= \boldsymbol{0}, & \boldsymbol{x} &\in \{\boldsymbol{x} : 0 < x_1 < d, |x_2| = h/2\}. \end{aligned}$$

In the discrete elastic lattice

$$\sum_{\boldsymbol{q}\in\mathcal{C}(\boldsymbol{p})} \Big[B(\boldsymbol{q})\left\{\boldsymbol{u}(\boldsymbol{p}+\boldsymbol{q};t) - \boldsymbol{u}(\boldsymbol{p};t)\right\} - \frac{\alpha\ell}{2}\boldsymbol{b}(\boldsymbol{q})\left\{\Theta(\boldsymbol{p}+\boldsymbol{q};t) + \Theta(\boldsymbol{p};t)\right\}\Big] = \boldsymbol{0}, \qquad \boldsymbol{p} \in \Gamma,$$

$$\boldsymbol{u}(\boldsymbol{p};t) = \boldsymbol{0}, \qquad \boldsymbol{p} \in \gamma_h.$$

The heat conduction problem

In the continuum

$$\begin{aligned}
\kappa\Delta\theta(\boldsymbol{x}) &= i\omega\theta(\boldsymbol{x}), && \boldsymbol{x}\in\Omega,\\
\theta(\boldsymbol{x}) &= T_0, && \boldsymbol{x}\in\Omega\cap\{\boldsymbol{x}:x_1=0\},\\
\theta(\boldsymbol{x}) &= 0, && \boldsymbol{x}\in\Omega\cap\{\boldsymbol{x}:x_1=d\},\\
\nabla[\theta(\boldsymbol{x})]\cdot\boldsymbol{p} &= 0, && \boldsymbol{x}\in\Omega\cap\{\boldsymbol{x}:|x_2|=h/2\}.
\end{aligned}$$

In the discrete lattice

$$\begin{aligned}
\vartheta(\boldsymbol{p}) &= \frac{1}{i\omega\Xi+|\mathcal{N}(\boldsymbol{p})|}\sum_{\boldsymbol{q}\in\mathcal{N}(\boldsymbol{p})}\vartheta(\boldsymbol{p}+\boldsymbol{q}), && \boldsymbol{p}\in\Gamma,\\
\vartheta(\boldsymbol{p}) &= T_0, && \boldsymbol{p}\in\gamma_0,\\
\vartheta(\boldsymbol{p}) &= 0, && \boldsymbol{p}\in\gamma_d,\\
\vartheta(\boldsymbol{p}) &= \frac{1}{|\mathcal{N}(\boldsymbol{p})|}\sum_{\boldsymbol{q}\in\mathcal{N}(\boldsymbol{p})}\vartheta(\boldsymbol{p}+\boldsymbol{q}), && \boldsymbol{p}\in\gamma_h.
\end{aligned}$$

An effective stress intensity factor for the lattice

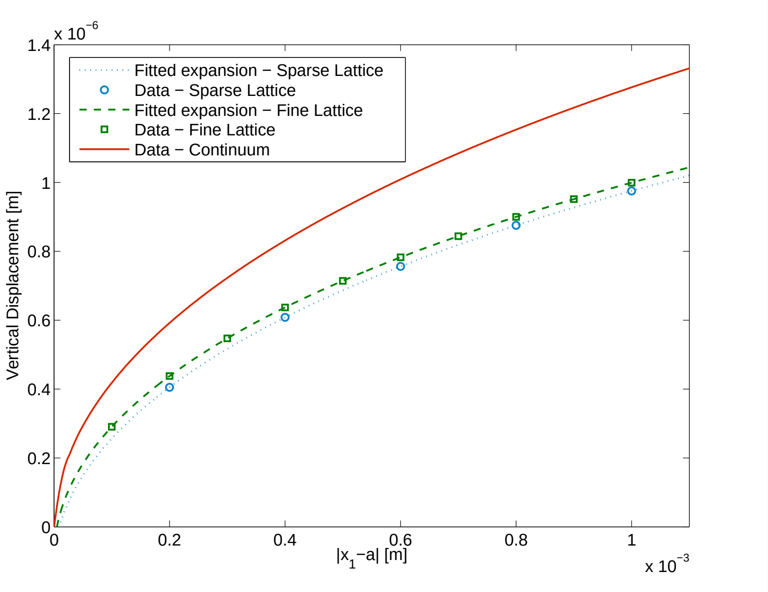
For a sufficiently refined lattice the vertical displacements behind the crack tip exhibit similar asymptotic behaviour to the continuum.

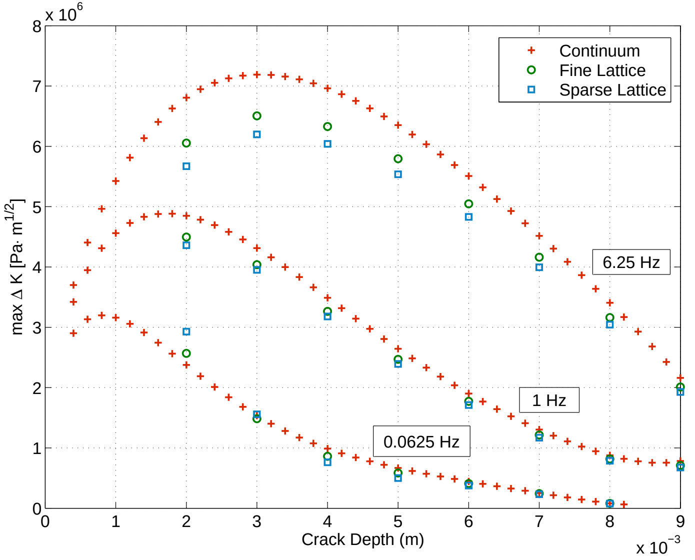
$$u_2(\boldsymbol{p}) \sim \frac{K_I^{(\text{eff})}}{(1-k^2)\mu}\sqrt{\frac{a - x_1(\boldsymbol{p})}{2\pi}} + b_1\left[a - x_1(\boldsymbol{p})\right] + b_2\left[a - x_1(\boldsymbol{p})\right]^{3/2} + b_3\left[a - x_1(\boldsymbol{p})\right]^2,$$

where $k = 3 - 4\nu$ and μ is the shear modulus corresponding to the homogenised continuum.

For the continuum, it is convenient to use a modified J-integral to compute the Stress Intensity Factor.

$$\frac{1-\nu^2}{E}K_I^2 = \frac{1}{2}\int_\gamma \left(\sigma_{ij}\epsilon_{ij} - \frac{E\zeta}{1-2\nu}T\epsilon_{ii}\right)\mathrm{d}x_2 - \int_\gamma \sigma_{ij}n_j\frac{\partial U_i}{\partial x_1}\mathrm{d}s + \frac{E\zeta}{1-2\nu}\int_\Gamma \epsilon_{ii}\frac{\partial T}{\partial x_1}\mathrm{d}A$$


x 10^−6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Vertical Displacement [m]
Fitted expansion − Sparse Lattice
Data − Sparse Lattice
Fitted expansion − Fine Lattice
Data − Fine Lattice
Data − Continuum
0
0.2
0.4
0.6
0.8
1
|x1−a| [m]
x 10^−3


x 10^6
max Δ K [Pa· m^1/2]
Crack Depth (m)
x 10^-3
Continuum
Fine Lattice
Sparse Lattice
6.25 Hz
1 Hz
0.0625 Hz
0
1
2
3
4
5
6
7
8
9