Dynamic response of a lattice with an inclusion.
Localised defect modes and homogenisation.

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We present a semi-analytical model of a dynamic response of a structured heterogenous solid with an "inclusion" represented as an array of masses embedded into a lattice. Special attention is given to a dynamic anisotropic response of the lattice and localised vibrations of the inclusion within a specially identified frequency range. Alongside the analysis of the lattice with a finite-size inclusion, we consider a waveguide problem for an infinite chain of masses embedded in a heterogeneous periodic lattice. Localisation and dispersion properties of waves propagating along such a waveguide have been analysed in detail. Furthermore, for frequencies close to the pass band upper boundary an analytical homogenisation approximation has been derived for localised waveforms within a finite-size inclusion for the case when the number of masses, which constitute the inclusion becomes large. Further extensions are in analysis of the boundary layer near the edges of the discrete inclusion in a lattice system.

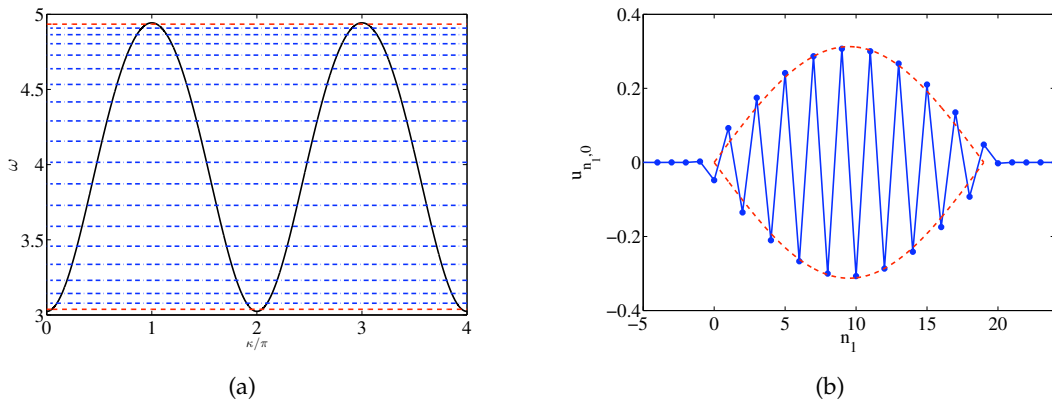(a)   (b)

Figure 1: (a) The dispersion curve for the infinite waveguide problem (solid line) together with the eigenfrequencies of the finite defect (dot-dash lines). The maximum and minimum eigenfrequencies are indicated by the dashed lines. (b) The highest eigenmode of the finite inclusion (solid line) together with the envelope (dashed line) given by the homogenisation approximation.