Multiscale asymptotic modelling of the size-scale effect in micro- and nano-indentation of a fibril-reinforced material

Ivan I. Argatov

Aberystwyth University, Ceredigion SY23 3BZ, Wales, UK
iva1@aber.ac.uk

Summary: A multiscale asymptotic approach is employed for mathematical modelling of indentation stiffness measurements for articular cartilage at the micro- and nano-scales. The present study is aimed to explain striking differences in the relation of the indentation microstiffness to the indentation nanostiffnesses experimentally observed for authentic cartilage and a model gel-microfiber composite.

Introduction

Indentation tests have been used for decades to identify mechanical properties of biological materials such as articular cartilage [1] and to assess its viability [2,3]. However, a number of papers have reported that indentation stiffness measurements in the millimetre-scale are not sensitive to even substantial changes in articular cartilage structure associated with early-stage osteoarthritis. It was experimentally observed [4] that all morphological and biomechanical changes occurring at the onset of osteoarthritis can solely be depicted on the nano-meter scale when probed by indentation-type atomic force microscopy (IT-AFM).

Biomechanical problem formulation

Articular cartilage exhibits scale-dependent indentation stiffness. In particular, as it was shown in [5], a micrometer-size spherical indenter revealed a unimodal stiffness distribution with the corresponding microstiffness of 1.3 MPa, whereas indenting articular cartilage with a nonometer-size pyramidal tip resulted in a bimodal nanostiffness distribution with the corresponding nanostiffnesses of 22.3 kPa and 384 kPa. In the context of exploring indentation treatment modalities for osteoarthritis, it is assumed [4] that the lower nano-stiffness peak reflects the stiffness of the proteoglycan gel, whereas the higher nano-stiffness peak reflects the stiffness of the collagen fibrils. Thus, a nonometer-size indenter tip can assess the indentation stiffness of articular cartilage at the level of the fibrils as well as at the level of the proteoglycan gel.

It should be emphasized [5] that single collagen fibrils exhibit stiffness in tension and indentation of a few gigapascals, while the IT-AFM values reported in [4,5] are much lower. In contrast, the mechanical behaviour of aggrecan gels show stiffness of only about 1 kPa, while the IT-AFM values reported in [5] are higher up to of order of magnitude. Thus, as it was observed in [5], indentation measurements of the isolated components do not take into account their mechanical behaviour within the tissue. In this study, a multiscale asymptotic approach is employed for mathematical modelling of indentation stiffness measurements for articular cartilage at the micro- and nano-scales.

Asymptotic modelling approach

We develop a simple asymptotic model to describe the mechanical behaviour of a relatively stiff infinite elastic fibre within an elastic matrix. Our asymptotic method is based on the method of matched asymptotic expansions and the asymptotic analysis of the so-called [6] junctions problems for elastic bodies pierced with elastic bars [7,8]. The key feature of the applied asymptotic method consists in with the modification of the asymptotic matching procedure suggested in [9] for elastic contact problems.

Conclusion

Asymptotic modelling (AM) is an analytical technique, which allows simplifying mathematical models in certain limit situations (thin inclusions, high relative stiffness, etc.). AM was successfully used for describing the interaction effect between cracks and inclusions [10], the size-effect in nanoindentation [11], and the interaction effect between asperities [12]. It is assumed that AM can explain striking differences in the relation of the indentation microstiffness to the indentation nanostiffnesses observed in [5] for authentic articular cartilage and a model gel-microfiber composite. In particular, it was found that the microstiffness of the model material amounted to only 0.66% of the articular cartilage microstiffness, whereas the model material's nanostiffnesses were found within the same order of magnitude as that of articular cartilage for its both phases. This observation was explained [5] by the fact that in the model material, the fibrils are neither cross-linked nor spatially stabilized by any strong interactions.

Acknowledgment

The financial support from the European Union Seventh Framework Programme under contract number PIIF-GA-2009-253055 is gratefully acknowledged.

References

[1] J. Jurvelin, I. Kiviranta, J. Arokoski, M. Tammi, H.J. Helminen, Indentation study of the biomechanical properties of articular cartilage in the canine knee. Eng. Med. 17:15–22, 1987.

[2] N.D. Broom and R. Flachsmann. Physical indicators of cartilage health: The relevance of compliance, thickness, swelling and fibrillar texture. *J. Anat.*, 202:481–94, 2003.

[3] R.K. Korhonen, S. Saarakkala, J. Töyräs, M.S. Laasanen, I. Kiviranta and J.S. Jurvelin. Experimental and numerical validation for the novel configuration of an arthroscopic indentation instrument. *Phys. Med. Biol.*, 48:1565–76, 2003.

[4] M. Stolz, R. Gottardi, R. Raiteri, S. Miot, I. Martin, R. Imer, U. Staufer, A. Raducanu, M. Duggelin, W. Baschong, A.U. Daniels, N.F. Friederich, A. Aszodi, and U. Aebi. Early detection of aging cartilage and osteoarthritis in mice and patient samples using atomic force microscopy. *Nat. Nanotechnol*, 4:186–92, 2009.

[5] M. Loparic, D. Wirz, A.U. Daniels, R. Raiteri, M.R. VanLandingham, G. Guex, I. Martin, U. Aebi, and M. Stolz. Micro- and nanomechanical analysis of articular cartilage by indentation-type atomic force microscopy: validation with a gel-microfiber composite. *Biophys. J.*, 98:2731–40, 2010.

[6] D. Leguillon and E. Sanchez-Palencia. Approximation of a two dimensional problem of junctions. *Comput. Mech.*, 6:435–55, 1990.

[7] I.I. Argatov and S.A. Nazarov. Equilibrium of an elastic body pierced by horizontal thin elastic bars. *J. Appl. Mech. Techn. Phys.*, 40:763–769, 1999.

[8] I.I. Argatov and S.A. Nazarov. Asymptotic analysis of problems in junctions of domains of different limit dimensions. An elastic body, pierced by thin rods. *J. Math. Sci.*, 102: 4349–87, 2000.

[9] I.I. Argatov. Pressure of a narrow band-like punch upon an elastic half-space. *Int. Appl. Mechanics*, 36:1363–8, 2000.

[10] A.B. Movchan, S.A. Nazarov and O.R. Polyakova. The quasi-static growth of a semi-infinite crack in a plane containing small defects. *Comptes rendus de l'Académie des sciences. Série 2, Mécanique*, 313:1223–8, 1991.

[11] I. Argatov. Frictionless and adhesive nanoindentation: Asymptotic modeling of size effects. *Mech. Materials*, 42, 807–15, 2010.

[12] I.I. Argatov. Electrical contact resistance, thermal contact conductance and elastic incremental stiffness for a cluster of microcontacts: Asymptotic modelling. *Q. J. Mechanics Appl. Math.*, 64: 1–24, 2011.