Mathematical Modelling of Interfacial Cracks in Ansitoropic and Piezoelectric Bimaterials

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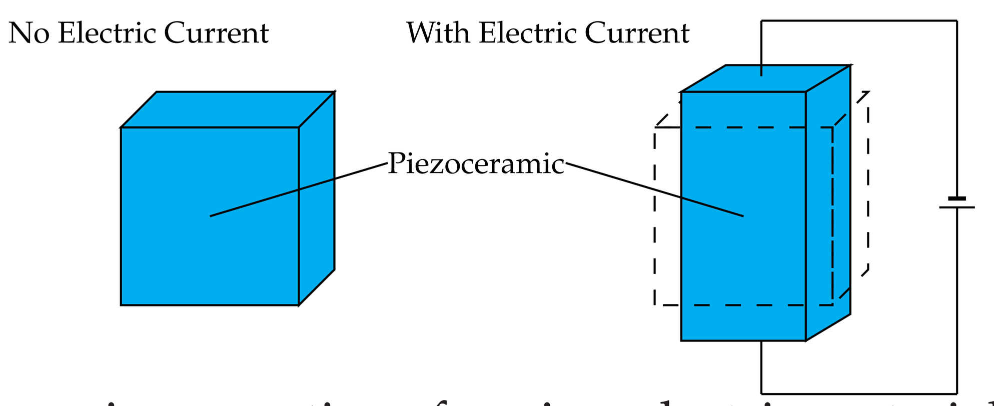
Piezoelectric materials

Piezoelectric materials generate an electric charge when subjected to mechanical stress and also deform when an electric current is run through them.



Governing equations for piezoelectric materials:

$$\sigma_{ij,i} = 0, \quad D_{i,i} = 0$$

where σ is the stress and D is the electrical induction. The elastic and electric fields are related in the following manner [1]

$$\sigma_{ij} = C_{ijrs}\varepsilon_{rs} - e_{sji}E_s,$$

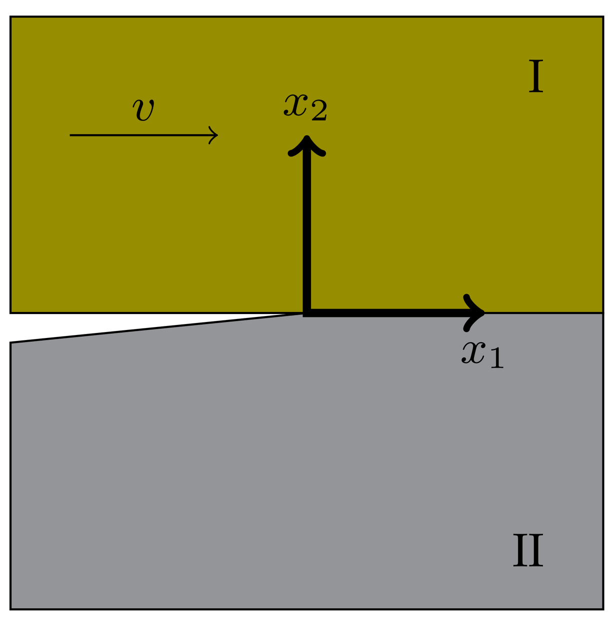
$$D_i = \omega_{is}E_s + e_{irs}\varepsilon_{rs},$$

where ε is strain, $\mathbf{C}$ is the material stiffness tensor, E is the electric field, ω is permittivity and e is the piezoelectric tensor. An eigenvalue problem was then developed in [1] to find solutions in the form of the extended displacement and traction fields:

$$\mathbf{U} = (\mathbf{u}^T, \Phi)^T, \quad \mathbf{t} = (\sigma_{12}, \sigma_{22}, \sigma_{32}, D_2)^T.$$

Problem formulation

Interfacial crack propagation in anisotropic bimaterials pose many interesting problems. The following geometry will be considered:



Perfect interface transmission conditions are considered:

- Continuation of displacements and tractions.
- Continuity of potential and charge for piezoelectric materials.

The following two examples are considered

1. Stress intensity factor analysis for a dynamic crack along the interface in an anisotropic bimaterial - no piezoelectricity [2]
2. Singular integral equations for a static interfacial crack between two piezoelectric materials [3]

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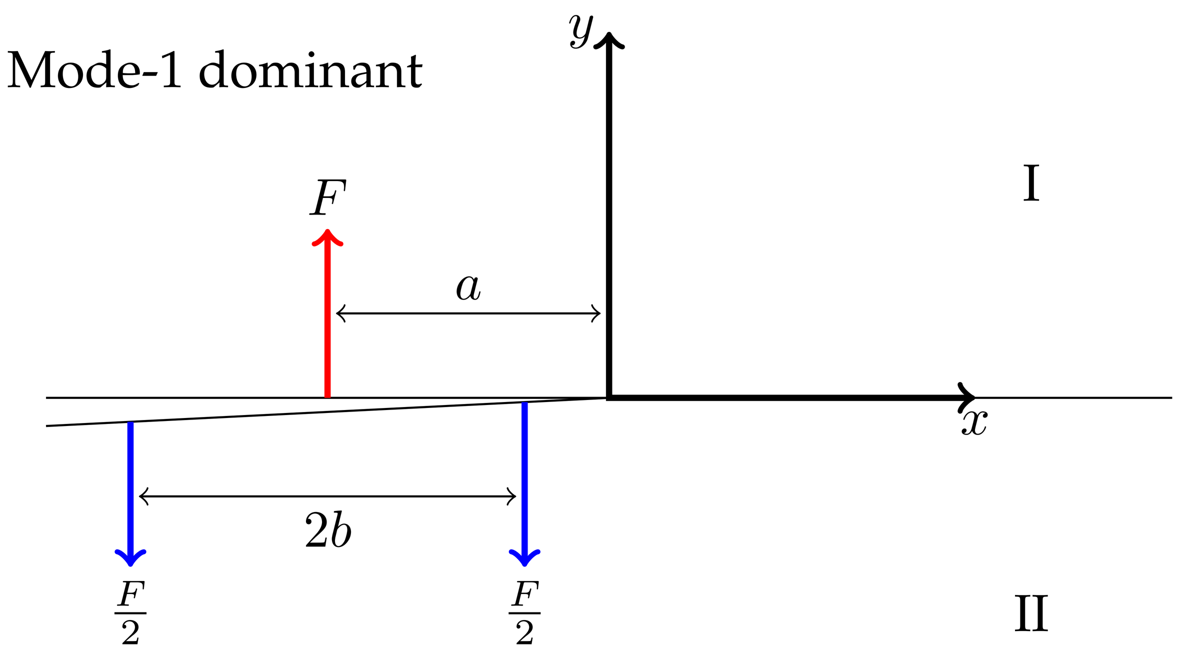
In-plane stress intensity factors for orthotropic bimaterials

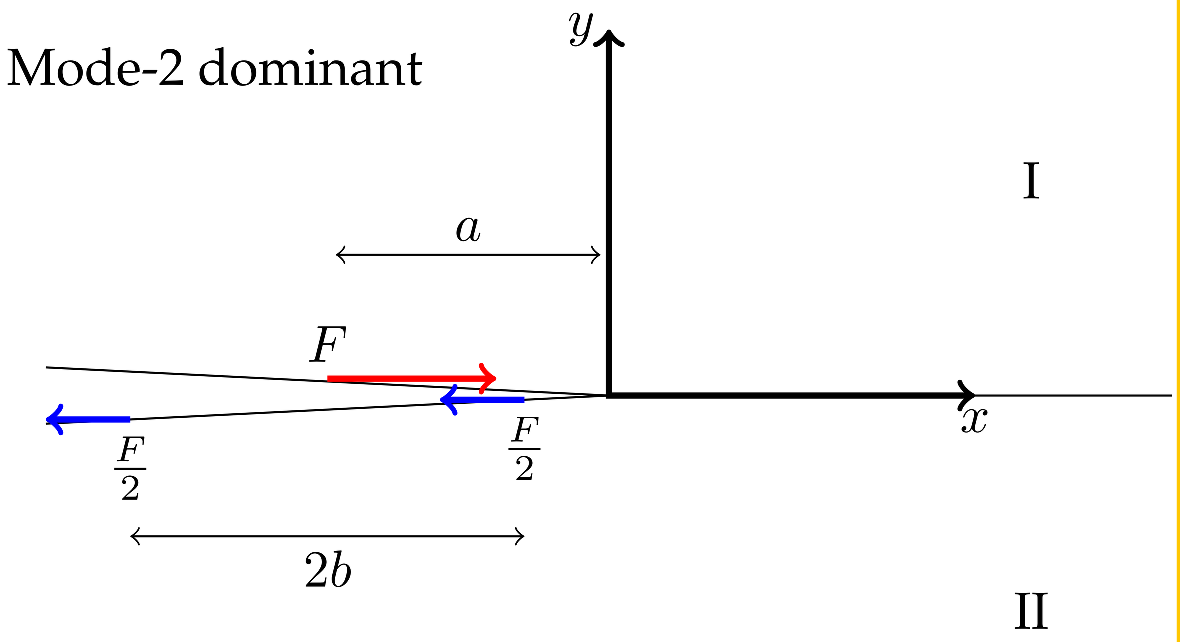
Mode-1 dominant

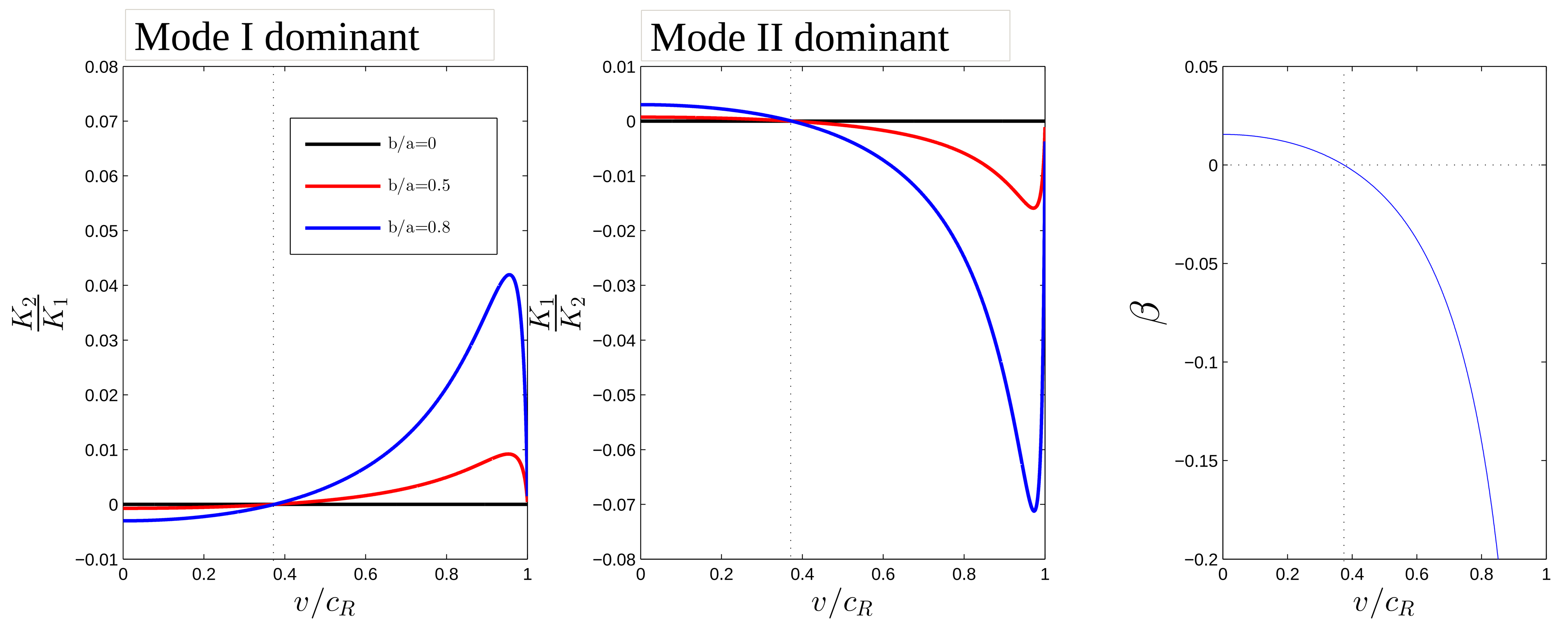


- Orthotropic materials used.
- New co-ordinates: $x = x_1 - vt, y = x_2$.
- Steady-state formulation now possible.
- Limiting velocity - Rayleigh wave speed of most compliant material, c_R.
- Two asymmetric loading configurations considered.

- Both loading systems are symmetric when $b = 0$.
- Increased values of b leads to a higher degree of asymmetry.
- Finding the stress intensity factor, $K = K_1 + iK_2$, which plays a key role in the first order asymptotics of displacements and tractions at the crack tip.

Mode-2 dominant





- Symmetric loading ($b/a = 0$) - stress intensity factor purely real for Mode-1 dominant loading or completely imaginary for Mode-2 dominant loading.
- Increased asymmetry leads to an increase in the non-dominant part of K - behaviour near c_R should be investigated practically.
- The Dundurs paramater, β, is real-valued and depends on the material paramaters and crack velocity, v.
- There exists a particular velocity, v_0, where $\beta(v_0) = 0$ - non-dominant part of K disappears at v_0 regardelss of loading asymmetry. Crack propagates straight along the interface. However, v_0 does not exist for every bimaterial.

Singular integral equations - out-of-plane piezoelectric effect

The example considered here has $v = 0$ and poling in the x_3 direction, therefore the in-plane fields are not affected by the piezoelectric effect and behave elastically. To study the piezo-effect, solutions are found in the form: $\mathbf{u} = (u_3, \phi)^T$, $\mathbf{t} = (\sigma_{32}, D_2)$.

Implementing weight functions and the Betti formula leads to the following singular integral equations:

$$\langle \mathbf{p} \rangle(x_1) + \mathbf{Y}[\![\mathbf{p}]\!](x_1) = -\frac{\mathbf{H}^{-1}}{\pi}\int_{-\infty}^{0} \frac{1}{x_1 - \eta}\frac{\partial [\![\mathbf{u}]\!]^{(-)}}{\partial \eta}\mathrm{d}\eta, \quad \text{for } x_1 < 0,$$

$$\mathbf{t}(x_1) = -\frac{\mathbf{H}^{-1}}{\pi}\int_{-\infty}^{0} \frac{1}{x_1 - \eta}\frac{\partial [\![\mathbf{u}]\!]^{(-)}}{\partial \eta}\mathrm{d}\eta, \quad \text{for } x_1 > 0,$$

where $\mathbf{H}$ and $\mathbf{Y}$ are bimaterial matrices depending on the material surface admittance tensors. $[\![\mathbf{p}]\!]$ and $\langle \mathbf{p} \rangle$ depend on the crack face loading, $\mathbf{p}$, in the following way:

$$[\![\mathbf{p}]\!](x_1) = \mathbf{p}^+(x_1) - \mathbf{p}^-(x_1), \quad \langle \mathbf{p} \rangle(x_1) = \frac{1}{2}\left[\mathbf{p}^+(x_1) + \mathbf{p}^-(x_1)\right].$$

References

[1] Suo, Z., Kuo, C. M., Barnett, D. M., Willis, J. R. Fracture mechanics for piezoelectric ceramics. *J. Mech. Phys. Solids* **40**, 739-765 (1992)

[2] Pryce, L., Morini, L., Mishuris, G. Weight function approach to a crack propagating along a bimaterial interface under arbitrary loading in an anisotropic solid. *JOMMS* **DOI: 10.2140/jomms.2013.8-8**, 479-500 (2013)

[3] Pryce, L., Morini, L. Weight functions and singular integral equations in piezoelectric bimaterials. *To be submitted*