Fracture waves propagating in discrete structures with non-local interactions

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Introduction

The crack propagation in ceramic materials limits their applicability. The classical technique of fracture analysis is based on the theory of linear elasticity and the solution for the crack problem is widely used by both scientists and engineers [1]. Nevertheless, this method does not allow to study the solution close to the crack tip and does not allow to study the dynamic effects of such fracture. The interfacial cracks, e.g. along ceramic/metal interfaces (fig.1), can be initiated by the lattices mismatch of the materials during the production stage [2]. In such a case a residual stress field can be generated along the interface which then can lead to the failure of a material.

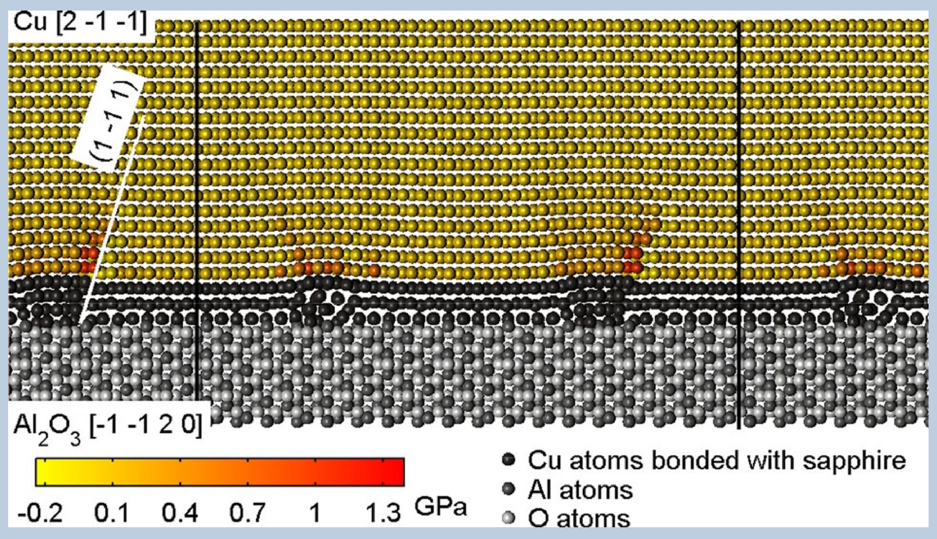

Figure 1: Distribution of elastic strain energy density in the copper section with the thickness Åat the center of the periodic cell [2].

Proposed model

We consider a crack propagation in a discrete structure with non-local interactions (fig.2) which can represent the prestressed interface. We assume: M is a mass of a particle, c_1 is a stiffness of the links between the particles and a substrate, c_2, c_3 is a spring constant between the closest and second-closest neighbors respectevely, n^* is a position of a crack tip which propagates with a constant speed v from left to right. We study the effect of introduced non-local interactions, i.e. the magnitude of c_3 in comparison with c_1 and c_2. The analysis is based mostly on the ideas proposed by Slepyan L.I. [3] and developed in various works, e.g. [4].

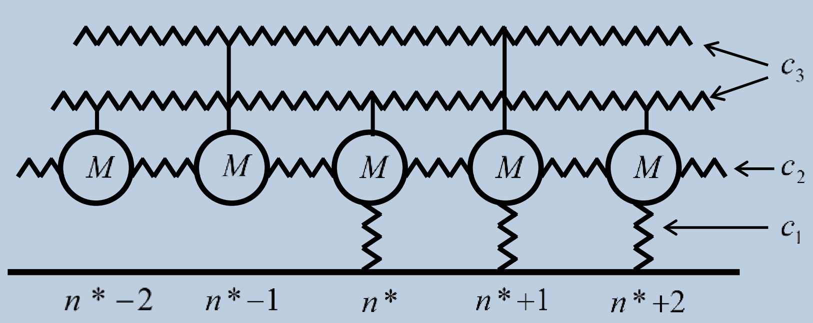

Figure 2: Discrete chain with non-local interactions.

References

[1] *Freund L. B.* Dynamic fracture mechanics.1990. Cambridge univ. press.

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[3] *Slepyan L.I.* Models and Phenomena in Fracture Mechanics. 2002. Springer,Berlin.

[4] *Mishuris G.S., Movchan A. B., Slepyan L. I.* Localised knife waves in a structured interface// Journal of Mechanics and Physics. 2009. Vol. 57. pp. 1958-1979.

Mathematical Formulation

Let us define the displacement of n-th particle as $u_n(t)$ and introduce the variable $\eta = n^* - vt$. The equation of motion for a displacement particle in terms of variable η takes form:

$$Mv^2u''(\eta) = c_3(u(\eta+2) + u(\eta-2) - 2u(\eta)) + c_2(u(\eta+1) + u(\eta-1) - 2u(\eta)) - c_1u(\eta)H(\eta), \quad (1)$$

where $H(\eta)$ is a Heaviside step function. The boundary conditions that we consider are:

- Constant load with magnitude C at $\eta = -\infty$.
- Constant load C with the background harmonic load $A_0\cos(k_f\eta - \phi)$ of amplitude A_0, frequency k_f and phase shift ϕ at $\eta = -\infty$.

The solution of the problem can be obtained by use of Fourier transform. This transform leads to a Wiener-Hopf type equation which can be solved by application of proper factorization procedure.

Results

The solution of problem (1) in terms of Fourier transform:

$$\begin{aligned} U_-(k) &= L_-(k)\left(\frac{C}{0+ik} + \frac{A}{0+i(k-k_f)} + \frac{\overline{A}}{0+i(k+k_f)}\right) \\ U_+(k) &= \frac{1}{L_+(k)}\left(\frac{C}{0-ik} + \frac{A}{0-i(k-k_f)} + \frac{\overline{A}}{0-i(k+k_f)}\right) \end{aligned} \quad (2)$$

The displacement field $u(\eta)$ can now be obtained from (2) by inverse Fourier transform.

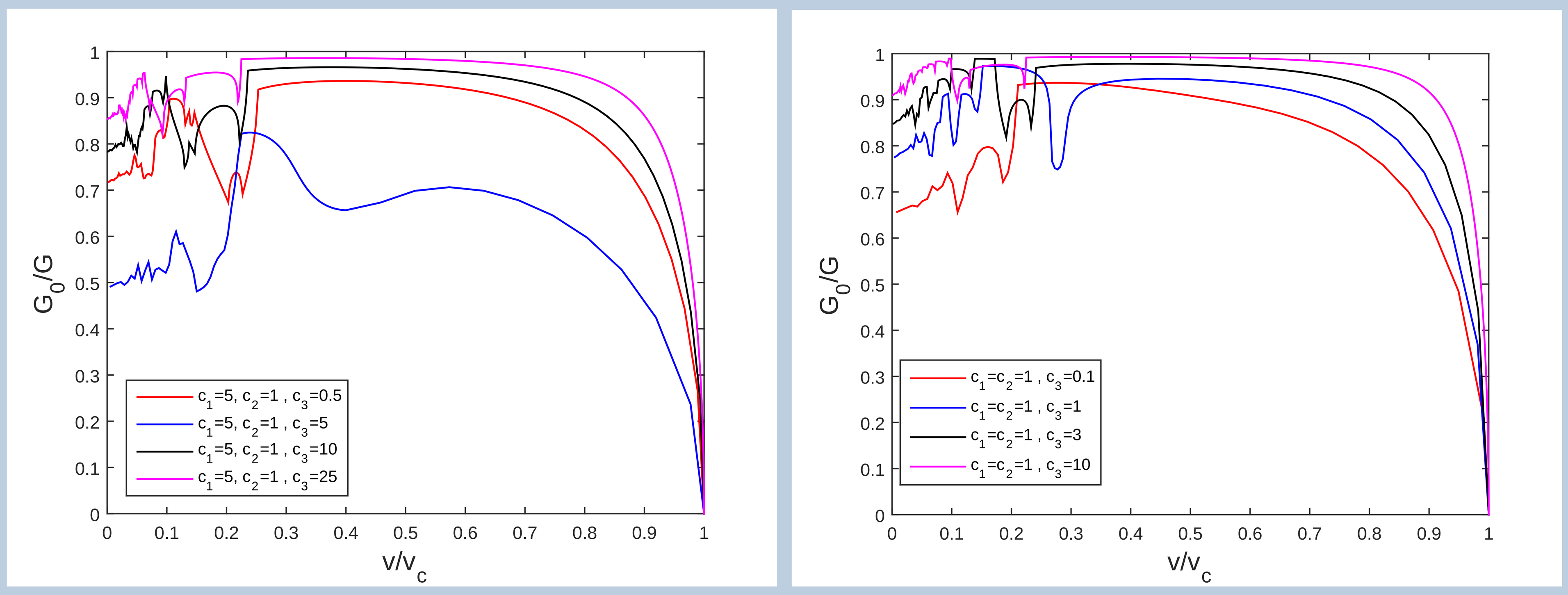

Figure 3: Dependence of energy ratio G_0/G on the ratio of crack speed v/v_c with different values of parametres of a model.

Expressions in (2) allow us to analyze the energetic characteristics of the fracture process in the discrete chain. From the asymptotic behavior of functions $U_\pm(k)$ at infinity we can get an expression for the energy release rate G. The plots on the fig.3 show the dependence of ratio of energies G_0/G on v/v_c, where G_0 is an accumulated energy in the spring before it breaks and v_c is the speed of sound of the "unbroken" part of chain. The loss of monotonicity of the dependence at low crack velocities reveal the specifics of the discrete model.

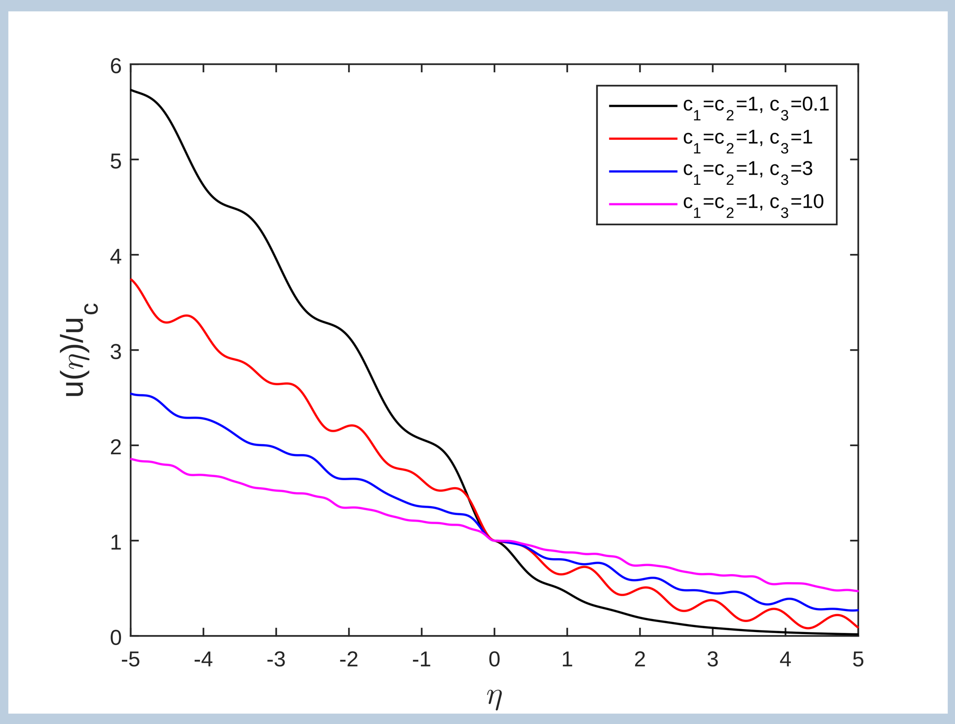

Figure 4: Dependence of a displacement field in the vicinity of a crack tip ($M = 1, v = 0.3$).

The dependence of function displacement field $u(\eta)/u_c$ is shown on fig.4, where we use a fracture criterion $u(\eta) = u_c$ as a condition of a spring failure. The plots show the waves running from the crack tip which demonstrate the dynamic effects of a crack propagation.

Conclusions

- Developed model allows to study the interfacial cracks in materials, e.g. along ceramic/metal interfaces.
- Introduced non-local interactions show the significant changes of a behavior of the system with a crack propagating at low speed.